and, of course,

$$\int^{h} 2\pi w(r,h) r \, dr = 1. \tag{4.4}$$

Several authors use a Gaussian function $\exp(-r^2/h^2)$ or a modified Gaussian function $\exp(-r^2/h^2) * (3/2 - r^2/h^2)$ [7, 15], but they are very smooth and they require a neighbours search over $3h$, so that we have chosen polynomial functions that are nonzero only in a finite domain. Then the summation over all particles is done only over the neighbouring particles. To reach a good interpolation accuracy, a minimum number of neighbours have to be taken into account, depending on the interpolation function. For the second and third degree polynomials w_2 and w_3 given below, a search over h is sufficient with respectively 25 and 21 neighbours, unlike the Gaussian function that requires 50 neighbours over $3h$:

$$\begin{aligned} W_2 = 18/(7\pi h^2) \quad & (1-3u^2) && \text{for} \quad u = r/h \leqslant \tfrac{1}{3} \\ & \tfrac{3}{4}(1-u^2) && \text{for} \quad \tfrac{1}{3} \leqslant u \leqslant 1 \\ & 0 && \text{for} \quad u > 1 \end{aligned} \tag{4.5}$$

$$\begin{aligned} W_3 = 40/(7\pi h^2) \quad & (1-6u^2+6u^3) && \text{for} \quad u = r/h \leqslant \tfrac{1}{2} \\ & 2(1-u)^3 && \text{for} \quad \tfrac{1}{2} \leqslant u \leqslant 1 \\ & 0 && \text{for} \quad u \geqslant 1 \end{aligned} \tag{4.6}$$

After testing these functions we have chosen W_3 which minimizes the number of neighbours and interpolates with errors not exceedig $O(h^2)$.

The Variable Kernel Size

One of the important points of our scheme lies in the variable kernel size h. Some authors [19, 20, 1, 14, 4] have used a variable h that allows a variable resolution, without the complex use of windows with varying cell size as in grid schemes. Moreover, the size of a fluid element adapts itself automatically according to the local density. The relationship between h and ρ is then [6]:

$$h^2 = K/\rho \qquad \text{where} \quad K \text{ is a constant.} \tag{4.7}$$

To optimize the accuracy of the calculations the following iterative procedure can be used:

$$h^2 = K/\rho \rightarrow \text{calculation of } w \rightarrow \text{calculation of } \rho$$

It converge for the K values that optimize the calculations. Another method can be used, such as the determination of the variance ($h^2 = \langle \Delta \mathbf{r}^2 \rangle - \langle \Delta \mathbf{r} \rangle^2$) over the