

and, of course,

$$\int_0^h 2\pi w(r, h) r dr = 1. \quad (4.4)$$

Several authors use a Gaussian function  $\exp(-r^2/h^2)$  or a modified Gaussian function  $\exp(-r^2/h^2) * (3/2 - r^2/h^2)$  [7, 15], but they are very smooth and they require a neighbours search over  $3h$ , so that we have chosen polynomial functions that are nonzero only in a finite domain. Then the summation over all particles is done only over the neighbouring particles. To reach a good interpolation accuracy, a minimum number of neighbours have to be taken into account, depending on the interpolation function. For the second and third degree polynomials  $w_2$  and  $w_3$  given below, a search over  $h$  is sufficient with respectively 25 and 21 neighbours, unlike the Gaussian function that requires 50 neighbours over  $3h$ :

$$W_2 = 18/(7\pi h^2) \quad \begin{array}{ll} (1 - 3u^2) & \text{for } u = r/h \leq \frac{1}{3} \\ \frac{3}{4}(1 - u^2) & \text{for } \frac{1}{3} \leq u \leq 1 \end{array} \quad (4.5)$$

$$0 \quad \text{for } u > 1$$

$$W_3 = 40/(7\pi h^2) \quad \begin{array}{ll} (1 - 6u^2 + 6u^3) & \text{for } u = r/h \leq \frac{1}{2} \\ 2(1 - u)^3 & \text{for } \frac{1}{2} \leq u \leq 1 \end{array} \quad (4.6)$$

$$0 \quad \text{for } u \geq 1$$

After testing these functions we have chosen  $W_3$  which minimizes the number of neighbours and interpolates with errors not exceeding  $O(h^2)$ .

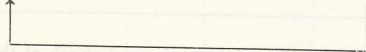
#### The Variable Kernel Size

One of the important points of our scheme lies in the variable kernel size  $h$ . Some authors [19, 20, 1, 14, 4] have used a variable  $h$  that allows a variable resolution, without the complex use of windows with varying cell size as in grid schemes. Moreover, the size of a fluid element adapts itself automatically according to the local density. The relationship between  $h$  and  $\rho$  is then [6]:

$$h^2 = K/\rho \quad \text{where } K \text{ is a constant.} \quad (4.7)$$

To optimize the accuracy of the calculations the following iterative procedure can be used:

$$h^2 = K/\rho \rightarrow \text{calculation of } w \rightarrow \text{calculation of } \rho$$



It converges for the  $K$  values that optimize the calculations. Another method can be used, such as the determination of the variance ( $h^2 = \langle \Delta r^2 \rangle - \langle \Delta r \rangle^2$ ) over the