

where  $\alpha$  is a constant,  $c$  is the sound speed,  $\mathbf{v}$  is the fluid velocity, and  $\rho$  is the density.

The Gingold–Monaghan (GM) viscous tensor: (4.10)

$$\Pi_{ij} = -c_{ij}^2/\rho_{ij}(\alpha_1 q_{ij} + \alpha_2 q_{ij}^2)$$

with

$$\begin{aligned} q_{ij} &= r_{ij}v_{ij}/(h_{ij}c_{ij})/(r_{ij}^2/h_{ij}^2 + \beta) & \text{for } v_{ij}r_{ij} < 0 \\ q_{ij} &= 0 & \text{for } v_{ij}r_{ij} > 0, \end{aligned}$$

where  $0 < \beta \ll 1$  is a constant and for any function  $G$ ; we define

$$G_{ij} = (G_i + G_j)/2. \quad (4.11)$$

$\Pi_{ij}$  is such that

$$(\mathbf{F}_i)_{\text{visc}} = -\sum_j m_j \Pi_{ij} \nabla_i w(r_{ij}, h_j). \quad (4.12)$$

We then consider an isothermal shock problem for a perfect gas analogous to that considered by Leboeuf *et al.* [11]. The initial state consists of a high density plateau surrounded by a region where the density is lower. The analytical solutions can be calculated only for 1D shocks so that we choose an initial symmetry—the density depends only on the  $x$  variable—that leads to a 1D solution, although the calculations are carried with a 2D code:

$$\begin{aligned} x < -7 \text{ or } x > 7 & \quad \rho = 4 \\ -7 < x < 7 & \quad \rho = 9. \end{aligned}$$

The results for each artificial viscosity are compared in Figs. 3a to 3f. The conclusion regarding the advantages of a given viscosity presents several differences from those with the shock tube problem (adiabatic) considered by Gingold and Monaghan [7] and Sod [18]. The first representation shows, for the case without artificial viscosity, the amplification of the oscillations that can lead to numerical divergences. The problem is solved by artificial viscosity as is shown by the other figures. For this type of shock the NR viscosity does not suppress all the oscillations. The density and velocity profiles are more smoothed by the B than by the GM, but the GM leads systematically to underestimated velocity values. The linear combination is comparable to the GM for the smoothing and to the B for the accuracy of the density values. All tests are carried out with variable size kernels. The various profiles are given for  $y=0$ , but for finite  $y$  the results are comparable; i.e., there is no movement in the  $y$ -direction except in the neighbourhood of the boundaries. Moreover, the same calculation was performed with inversion of the  $x$  and  $y$  variables and the equivalence of the two directions was checked. We conclude