# Frequency Multipliers 

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## What is a Frequency Multiplier ?

## Definition :

A frequency multiplier of order N is an electronic device that converts an input sinusoidal signal of frequency $F_{1}$ and power $P_{1}$ to an output sinusoidal signal of frequency $\mathrm{F}_{\mathrm{N}}=\mathrm{N}_{\times} \mathrm{F}_{1}$ and power $\mathrm{P}_{\mathrm{N}}$.

In practice a frequency multiplier generates unwanted harmonics at frequencies $F_{k}=k \times F_{1}$ with $k \neq N$ and power $P_{k}$ with

$$
\sum_{k \neq 1, k \neq N}^{\infty} P_{k} \ll P_{N} \quad \text { (frequency multiplier) }
$$

A frequency multiplier is therefore different from a comb generator that generates a series of harmonics which power decreases (usually) with the increasing frequency :

$$
P_{k} \geq P_{k+1} \quad \text { (comb generator) }
$$

## What is a Frequency Multiplier ?

Frequency Multiplier :


Comb Generator :


Spectrum of input signal



Spectrum of output signal

## Definition :

A frequency multiplier of order 2, 3, 4, N is called respectively a DOUBLER, a TRIPLER, a QUADRUPLER, a N-UPLER

## Definition :

The conversion efficiency is the ratio $\eta=P_{N} / P_{1}$ (DC Power is not considered)
With 3-terminal devices (transistors) it is possible that $\eta \geq 1$. With 2-terminals devices (diodes) $\eta \leq 1$.

Maximum conversion efficiency of an ideal rectifier :
The conversion efficiency $\eta$ of an ideal rectifier is limited by $1 / N^{2}$ where $N$ is the order of multiplication. Therefore, purely resistive non-linear devices are not suited for fabricating high-efficiency frequency multipliers.

## Maximum efficiency of an ideal reactive non-linear device :

The maximum conversion efficiency of an ideal reactive non-linear device (pure varactor, with no-loss) is $\eta=1$, regardless of the order of multiplication $N$ (Penfield and Rafuse, 1962)

## Practical frequency multipliers :

Series resistances in the device and circuit losses strongly affect the conversion efficiency. Reactive devices give more conversion efficiency than resistive devices but are harder to match other a wide frequency range (it is more difficult to couple the input signal to the device and more difficult to extract the output signal from the device).

High order frequency multipliers are very difficult to built. At frequencies above 100 GHz , it is better to use a cascade of multipliers of lower order than a single high order multiplier.

## What is a Frequency Multiplier ?

## Conversion efficiency of a chain of multipliers :

For a chain $\otimes N_{1} \otimes N_{2}$ of two cascaded multipliers of respective order $N_{1}$ and $N_{2}$, the conversion efficiency of the chain is $\eta\left[N_{1}, N_{2}\right]$.
A high order frequency multiplier of order $N_{3}=N_{1} \times N_{2}$ has usually a conversion efficiency $\eta\left[N_{3}\right]<\eta\left[N_{1}, N_{2}\right]$.

The conversion efficiency of a multiplier depends on many parameters. At a given output frequency, order of multiplication and choice of device technology, the conversion efficiency depends on the input power. Consequently the efficiency of the chain $\otimes N_{1} \otimes N_{2}$ is not necessary the same as the efficiency of the chain $\otimes N_{2} \otimes N_{1}$ :

$$
\eta\left[\mathrm{N}_{1}, \mathrm{~N}_{2}\right] \neq \eta\left[\mathrm{N}_{2}, \mathrm{~N}_{1}\right]
$$

It is common to write $\eta\left[N_{1}, N_{2}\right]=\eta\left[N_{1}\right] \times \eta\left[N_{2}\right]$ but this relation is valid only if the multipliers are isolated (no reflected power by the second multiplier at its input port can reach the first multiplier through its output port).

## Example: 1.9THz Local Oscilator Chain for HIFI (HERSCHEL)



## Why do we need frequency multipliers?

$\checkmark \quad$ Frequency multipliers are used to synthesize sinusoidal signals every time it is easier / cheaper to use a low frequency fundamental source cascaded with a frequency multiplier rather than using directly a fundamental source at the desired frequency.

Examples, frequency multipliers are used for:
$>$ building ultra-stable sources at high frequencies using the reference signal given by a quartz / atomic clock
> generating signals where there is NO solid-state fundamental sources (filling the THz gap)

## Noise issues

$\checkmark \quad$ A frequency multiplier degrades the phase-noise of the fundamental source by a factor $20 \log (N)$, where $N$ is the multiplication factor.
$\checkmark \quad$ A frequency multiplier adds amplitude modulation noise (AM), which power depends on the quality of the bias, the design and the fabrication of the circuit. In many cases AM noise is not significant compared to the additional phase noise introduced by frequency multiplication.

spacecraft



- LO for space-borne and groundbased heterodyne receivers:
- electronically tune-able submillimeter and THz sources
- Reliability / size / power consumption / temperature


## Solid-State THz Sources (CW) State-Of-The-Art



## Solid-State THz Sources (CW) below 2.5 THz



Compiled by Peter Siegel, 2003, updated by Alain Maestrini, 2005

Non linear-device pumped with a sinusoidal signal :


$$
\left\{\begin{array}{l}
I_{d}(V)=a_{0}+a_{1} V_{d}+a_{2} V_{d}^{2} \\
V_{g}=R_{g} \cdot I_{d} \\
V_{d}=V_{1}-V_{g}=v_{1} \cos \left(\omega_{1} t\right)-R_{g}\left(a_{0}+a_{1} V_{d}+a_{2} V_{d}^{2}\right)
\end{array}\right.
$$

These equations are true $\forall \mathrm{t}$. When the permanent regime is reached :
$V_{d}(t)=\sum_{n \in \mathbb{N}} V_{n} \cos \left(n \cdot \omega_{1} \cdot t+\varphi_{n}\right)$

## Devices for THz Frequency Multipliers

There are mainly two device technologies in competition for THz frequency multipliers :
$>$ Heterostructure Barier Varactor are suited for the generation of odd harmonics due to their internal symmetry:

| Epitaxial layer of IEMN HBVs |  |  |  |
| :--- | :--- | :--- | :---: |
| $\qquad$InGaAs $5 \times 10^{18} \mathrm{~cm}^{-3}$ 500 nm  <br> InGaAs $1 \times 10^{17} \mathrm{~cm}^{-3}$ 300 nm  <br> InGaAs Undoped 5 nm  <br> InAlAs Undoped 5 nm  <br> AlAs Undoped 3 nm  <br> InAlAs Undoped 5 nm  <br> InGaAs Undoped 5 nm  <br> InGaAs $1 \times 10^{17} \mathrm{~cm}^{-3}$ 300 nm  <br> InGaAs $5 \times 10^{18} \mathrm{~cm}^{-3}$ 500 nm  <br> InP Substrate    <br>     |  |  |  |



## Devices for <br> Devices for THz Frequency Multipliers

> Schottky diodes are the simplest possible devices: a metal deposited on a doped semiconductor.


Epitaxial layer of a Schottky diode
$I(V)$ curves of a Schottky diode


## Devices for <br> THz Frequency Multipliers

$>$ Schottky diodes are very fast devices and are (still !) the best devices for high efficiency THz frequency multipliers.
$C_{j}(V)=C_{j}(0) \cdot \frac{1}{\sqrt{1-V / V_{b}}}$ for $V<V_{b}$
$\mathrm{V}_{\mathrm{b}}$ is the built-in voltage.
Usualy $\mathrm{Vb} \simeq 0.8 \mathrm{~V}$ to 0.9 V for GaAs diodes.
$\mathrm{C}_{\mathrm{j}}(0)$ is proportional to the anode area
(if edge effects are non-significant)
and depends on the doping.


## Planar Schottky diodes

## Surface channel discrete planar diode (Univ. Bath then UVa - late 80's)

## Advantages

- reliability (space application)
- integration (balanced designs)
- Reproducibility (wide band designs)

Anode contact pad
Finger

Semi-insulating substrate (GaAs intrinsic)


Active layer (GaAs $\mathrm{n}^{+}$)
Ohmic contact
$\mathrm{SiO}_{2}$ passivation layer

## Example 1 : 540-640 GHz balanced Tripler

- Topology : balanced design, idler tuned in a virtual loop




## JPL 4-anode 540-640 GHz Balanced Tripler (partial 3D view)

I'Obsejvatoire


## JPL 4-anode 540-640 GHz Balanced Tripler (3D view of bottom block)

I'Observatoire


## JPL 4-anode 540-640 GHz Balanced Tripler (picture of the chip with bias circuit)



## Why is the circuit balanced ?



Due to the symmetry of the circuit and the symmetry of the excitation (input signal) :
$i_{b}(t)=i_{a}\left(t+T_{1} / 2\right)$
where $T_{1}$ is the period of the input signal
$i_{a}(t)=\sum_{n-\infty}^{+\infty} a_{n} e^{j n \omega_{1} t}$
$i_{b}(t)=\sum_{n=-\infty}^{+\infty} a_{n} e^{i m \omega_{1}\left(t+T_{1} / 2\right)}$
where $\left(a_{n}\right)_{n \in \mathbb{Z}}$, are complex coefficients that depend on the circuit and the strength of the fundamental signal. With the following notations:

$$
\begin{align*}
& I_{\text {even } \mathrm{H}}=\sum_{n--\infty}^{+\infty} a_{2 n} e^{j 2 n \sigma_{1} t}  \tag{4}\\
& I_{\text {odd } \mathrm{H}}=\sum_{n-\infty}^{+\infty} a_{2 n+1} e^{j(2 n+1) \omega_{1} t} \tag{5}
\end{align*}
$$

equations (2) and (3) bocome :
$i_{a}(t)=I_{\text {even } \mathrm{H}}+I_{o d d \mathrm{H}}$
$i_{b}(t)=I_{\text {even } \mathrm{H}}-I_{\text {odd } \mathrm{H}}$

## Why is the circuit balanced ?

The currents $I_{\text {even } \mathrm{H}}$ and $I_{\text {odd } \mathrm{H}}$ contain respectively the currents at the second and the third harmonics. They are new sources that generate electromagnetic waves in the circuit. Because of the symmetry, the currents $I_{\text {even H }}$ generate two sets of electromagnetic waves flowing in opposite directions and defined respectively by the electric fields $\vec{E}_{\text {even } \mathrm{H}}^{-}$and the currents $i_{\text {even } \mathrm{H}}^{-}$and by the electric fields $\vec{E}_{\text {even } \mathrm{H}}^{+}$and the currents $i_{\text {even } \mathrm{H}}^{+}$. Both currents, $i_{\text {even } \mathrm{H}}^{-}$and $i_{\text {even } \mathrm{H}}^{+}$, consist of two components of the same magnitude but opposite sign flowing along the edges of the suspended microstrip line. On the other hand, the currents $I_{\text {odd } H}$ generate two sets of electromagnetic waves flowing in opposite directions and are defined respectively by the electric fields $\vec{E}_{\text {odd } H}^{-}$and the currents $i_{\text {odd } H}^{-}$and by the electric fields $\vec{E}_{\text {odd } H}^{+}$and the currents $i_{\text {odd } H}^{+}$. Both the currents $i_{\text {odd } H}^{-}$and $i_{\text {odd } H}^{+}$are divided into two components of the same magnitude and sign, flowing along the edges and the center of the suspended microstrip line.

## Why is the circuit balanced?

Consequently, if the dimensions of the circuit allow it, the electromagnetic waves generated by the currents $I_{\text {even } H}$ propagate in a TE mode along the suspended microstrip line, whereas the electromagnetic waves generated by the currents $I_{o d d H}$ propagate in a TEM mode, independently of the circuit dimensions. Therefore, to balance the circuit it is important to confine the second harmonic in a virtual loop, and the TE mode should be cut off at the idler frequency.*

The idler frequency for a frequency tripler is the second harmonic of the input signal

## Design Method : optimizing the diode cell



Optimizing the diode cell and anode area of the $540-640 \mathrm{GHz}$ tripler (doping=1E17cm³, Cj0 05.7 fF each)

# Design Method : optimizing the diode cell 

Picture of the multiplier


Optimizing the diode cell and anode area of the $540-640 \mathrm{GHz}$ tripler (doping $=1 \mathrm{E} 17 \mathrm{~cm}^{-3}, \mathrm{Cj} 0 \approx 5.7 \mathrm{fF}$ each)




## ADS simulations with S-parameters calculated with HFSS for each harmonic



Impact of CjO on the bandwidth of the multiplier and impact on the choice of circuit variations
(JPL 600 GHz balanced Tripler)
PERFORMANCE vs Cj0 :
Chips and circuits only differ by the size of the anodes

(JPL 600 GHz balanced Tripler)
PERFORMANCE vs Rs :
Chips and circuits only differ by the series resistance of the diodes


## Simulations vs Measurements @T=300K

Output Power and Efficiency at 300K of two 600 GHz Balanced Triplers


## Simulations vs Measurements @T=120K

Output Power and Efficiency at 120K of two 600 GHz Balanced Triplers


## Example 2 : <br> 1.9 THz balanced tripler biasless 3D view of bottom block



### 1.9THz balanced tripler biasless



### 1.9THz balanced tripler biasless

(detail of the diode area)


## JPL 1.55-1.7 THz and 1.7-1.9 THz Balanced Triplers





## Conclusions

$\checkmark$ Diode-based MMIC-like circuit in waveguide mount gives the best performance in terms of power and bandwidth at sub-millimeter wavelength.
$\checkmark$ Power and bandwidth of multiplier chains working in the 1 to 2 THz range is strongly limited by the power available in the $300-400 \mathrm{GHz}$ range
$\checkmark$ Number of anode per chip is limited: increase in performance will come from advanced power combining schemes.

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