21-cm signal from Reionization :
Astrophysical parameters reconstruction using Supervised Learning

Action Fédératrice

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Context

Models :

- Theoretical
- Numerical
- ...

Parameters :

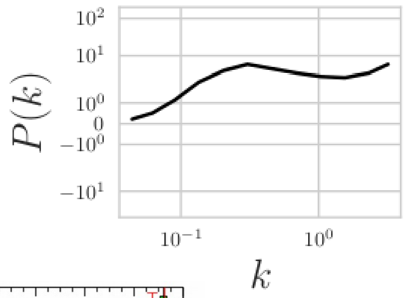
ζ_{ion}

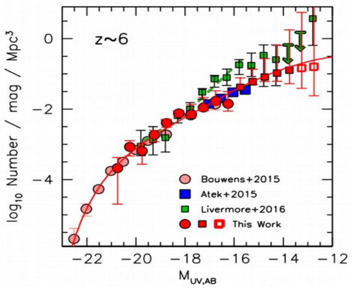
$M_{\rm min}$

R_{mfp}

T_{vir}

Observables :



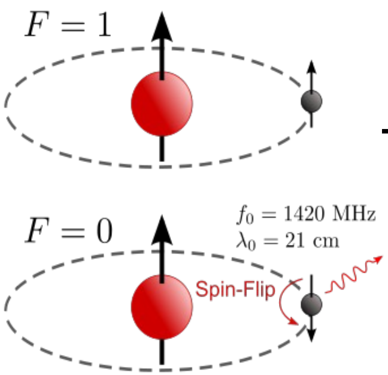


Bouwens et al. 2017

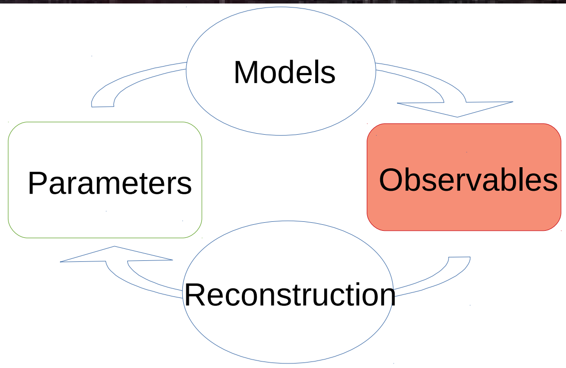
Reconstruction:

- Bayesian MCMC
- Supervised Learning
- ...

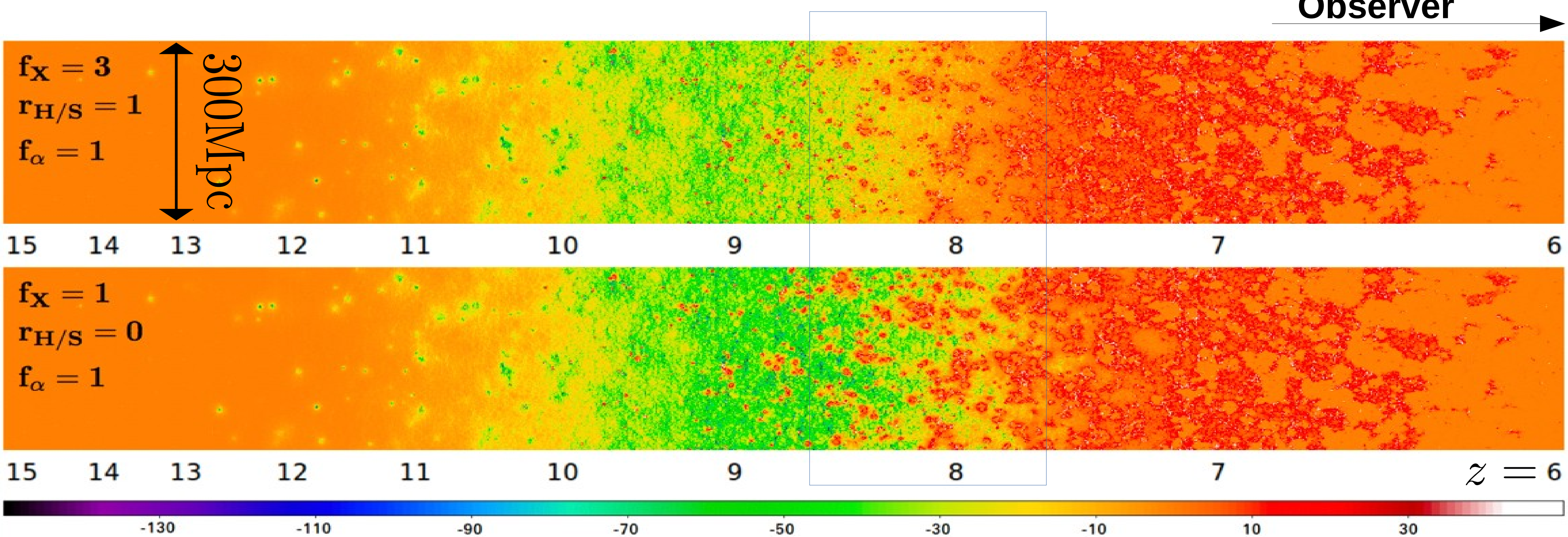
21cm Signal



- Determined by complex correlated physical effects



$$\delta T_B \propto 28(1+\delta)x_{\mathrm{HI}}\left(\frac{T_S - T_{CMB}}{T_S}\right)\left(1+\frac{1}{H}\frac{d\nu}{dr}\right)^{-1}\mathrm{mK}$$



Semelin et al. 2017

Semi-numerical code 21cmFAST

Mesinger et al. 2011

21cmFast

Parameters :

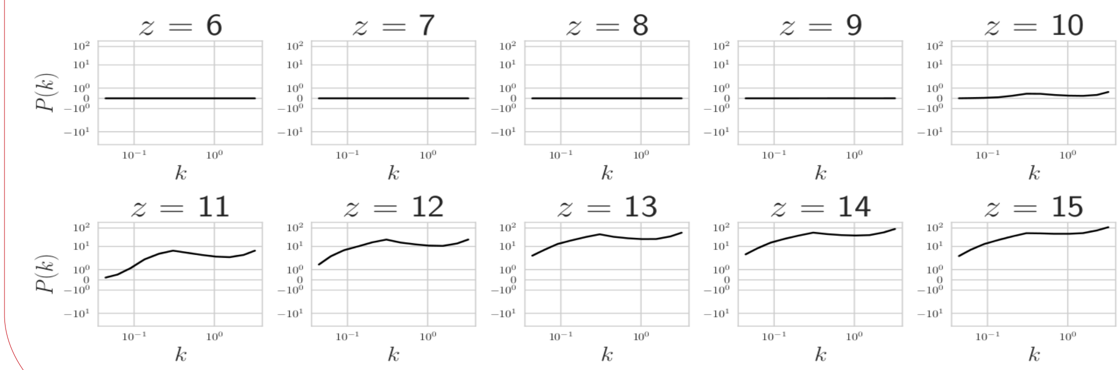
Ionizing emissivity ζ_{ion}

Lyman-Limit system influence R_{mfp}

Minimal mass of proto-galaxies T_{vir}

Observables :

21cm power spectrum



Neural Networks

Parameters reconstruction using supervised learning

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Analysing the 21 cm signal from the epoch of reionization with artificial neural networks

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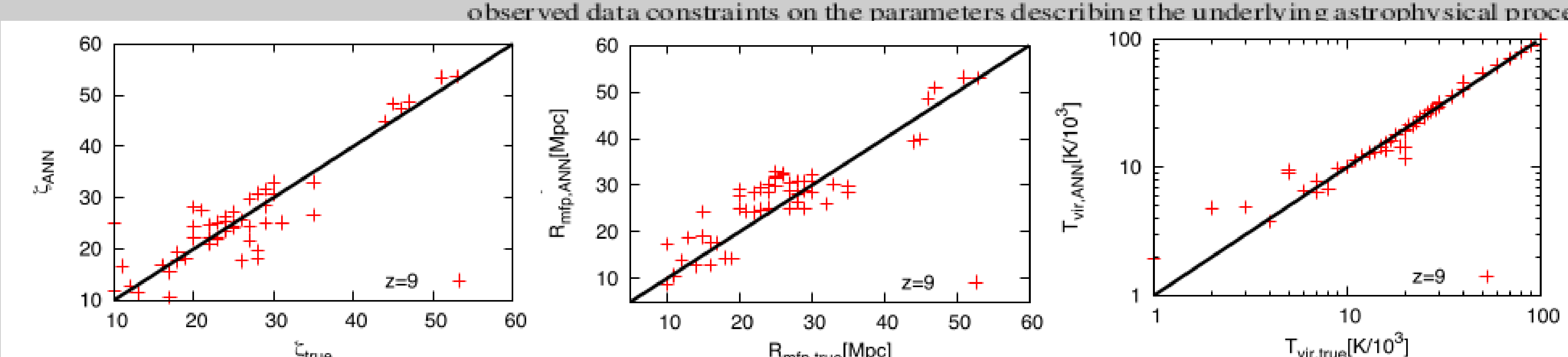
Sorbonne Universités, UPMC, LERMA, Observatoire d

Accepted 2017 March 22. Received 2017 March 22; in

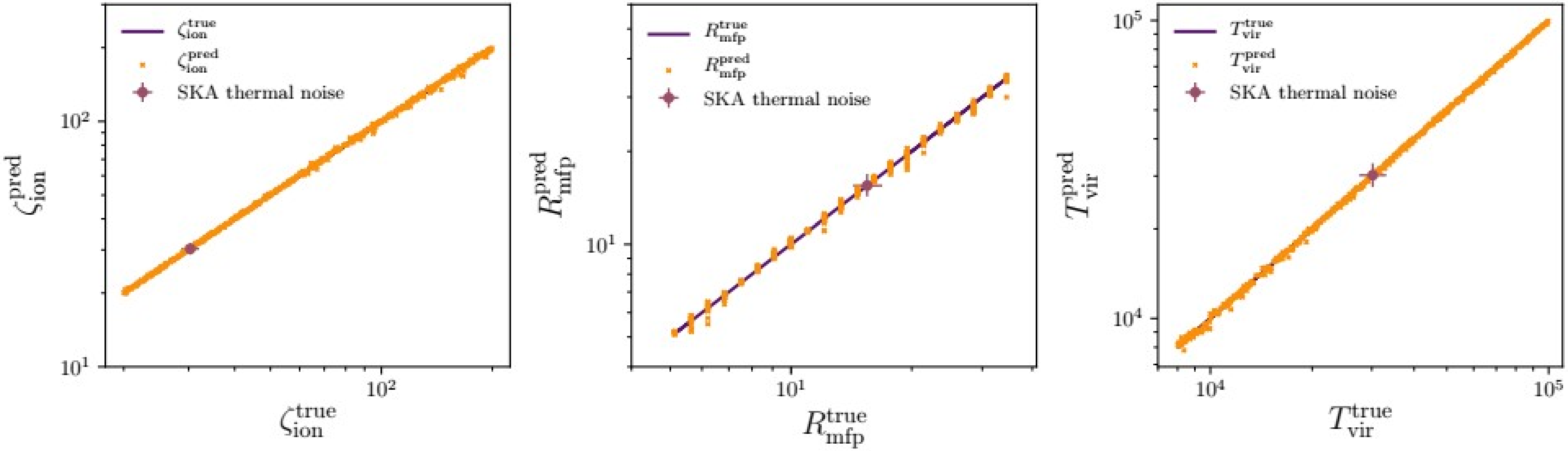
	$\chi_{\rm wo/noise}$	$\chi_{\rm w/noise}$	$\chi_{\rm w/noise,zevolution}$	$\chi_{\rm w/noise,reduced}$
$R_{\rm mfp}$	0.228	0.258	0.172	0.262
ζ	0.271	0.288	0.168	0.290
$\log(T_{\rm vir})$	0.027	0.038	0.019	0.029

ABSTRA
The 21 cm
While a sir
ers, the SKA
observed data constraints on the parameters describing the underlying astrophysical processes,
the quality of the reconstruction and that using the power spectrum at several redshifts as an



Improved Neural network

- 14 vs 80 neurons, 70 vs 2400 points in the learning samples, redshift evolution
- Better determination of the learning parameters



	$\chi^2_{\rm wo/noise}$
$R_{\rm mfp}$	0.228
ζ	0.271
$\log(T_{\rm vir})$	0.027

×10 accuracy

	χ
$R_{\rm mfp}$	0.02526
$\zeta_{\rm ion}$	0.01369
$\log(T_{\rm vir})$	0.00154

Linear Regression

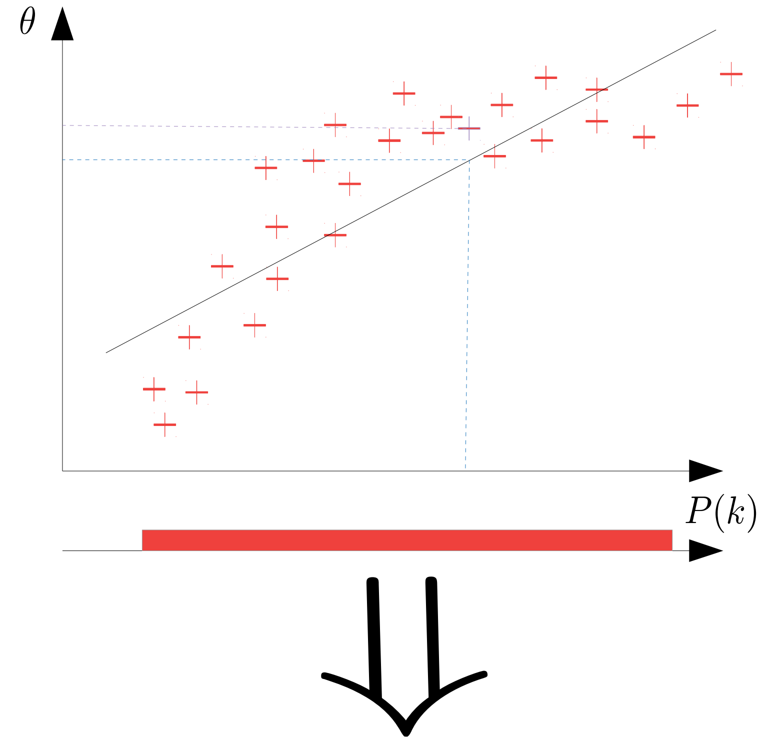
- Another supervised learning method
- Minimization of :

$P(k_1, z_1), ..., P(k_m, z_n)$

$$\min_{\alpha,\beta_j} \sum_{i=1}^{N_{\text{Sample}}} \left(y_i - (\alpha + \sum_{j=1}^{N_{\text{dim}}} \beta_j x_{i,j}) \right)^2$$

$T_{vir}, \zeta_{ion}, ..$

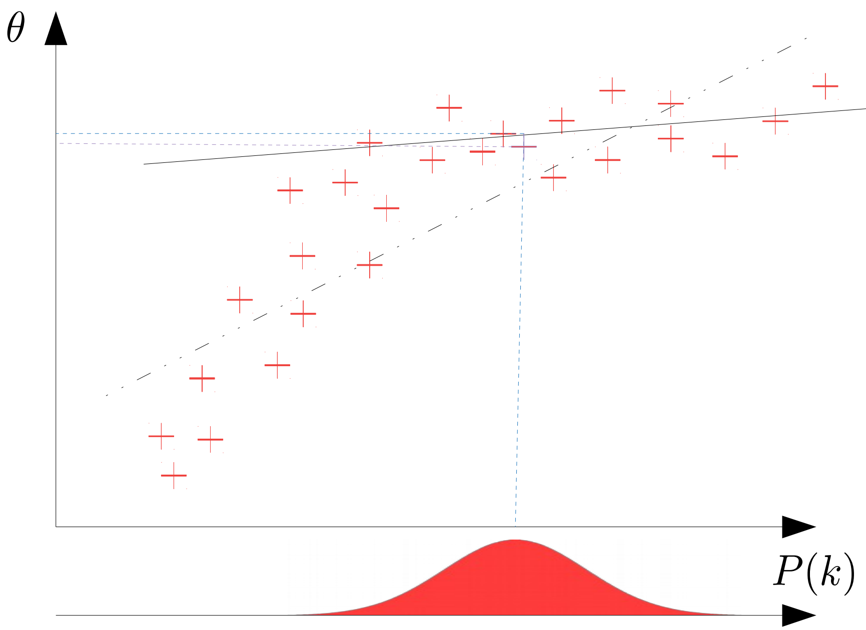
	ζ_{ion}	R_{mfp}	$log\,(T_{\text{vir}})$
$\chi_{\text{Shimabukuro}}$	0.271	0.228	0.027
$\chi_{\text{Neural Network}}$	0.01369	0.02526	0.00154
$\chi_{\text{Least Square}}$	0.01816	0.07996	0.00294



120 dimensions !

Kernel Regression

$$\min_{\alpha,\beta_j} \sum_{i=1}^{N_{\text{Sample}}} K_\sigma(x_0, x_i) \left(y_i - (\alpha + \sum_{j=1}^{N_{\text{dim}}} \beta_j x_{i,j}) \right)^2$$



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$\chi_{\text{Least Square}}$	0.01816	0.07996	0.00294
$\chi_{\text{Kernel Regression}}$	0.01192	0.06129	0.00275

Ridge Regression with Kernel

$$\min_{\alpha,\beta_j}\left[\sum_{i=1}^{N_{\text{Sample}}} K_\sigma\left(x_0, x_i\right)\left(y_i - (\alpha + \sum_{j=1}^{N_{\text{dim}}} \beta_j x_{i,j})\right)^2 + \lambda \sum_{j=1}^{N_{\text{dim}}} \beta_j^2\right]$$

	ζ_{ion}	R_{mfp}	$log\left(T_{\text{vir}}\right)$
$\chi_{\text{Shimabukuro}}$	0.271	0.228	0.027
$\chi_{\text{Neural Network}}$	0.01369	0.02526	0.00154
$\chi_{\text{Least Square}}$	0.01816	0.07996	0.00294
$\chi_{\text{Kernel Regression}}$	0.01192	0.06129	0.00275
$\chi_{\text{Kernel+Ridge Regression}}$	0.00676	0.01756	0.00091

Local determination of the hyperparameters

$$\min_{\alpha,\beta_j,\sigma,\lambda}\left[\sum_{i=1}^{N_{\mathrm{Sample}}} K_\sigma(x_0,x_i)\left(y_i-(\alpha+\sum_{j=1}^{N_{\mathrm{dim}}}\beta_j x_{i,j})\right)^2+\lambda\sum_{j=1}^{N_{\mathrm{dim}}}\beta_j^2\right]$$

	ζ_{ion}	R_{mfp}	$log\left(T_{\mathrm{vir}}\right)$
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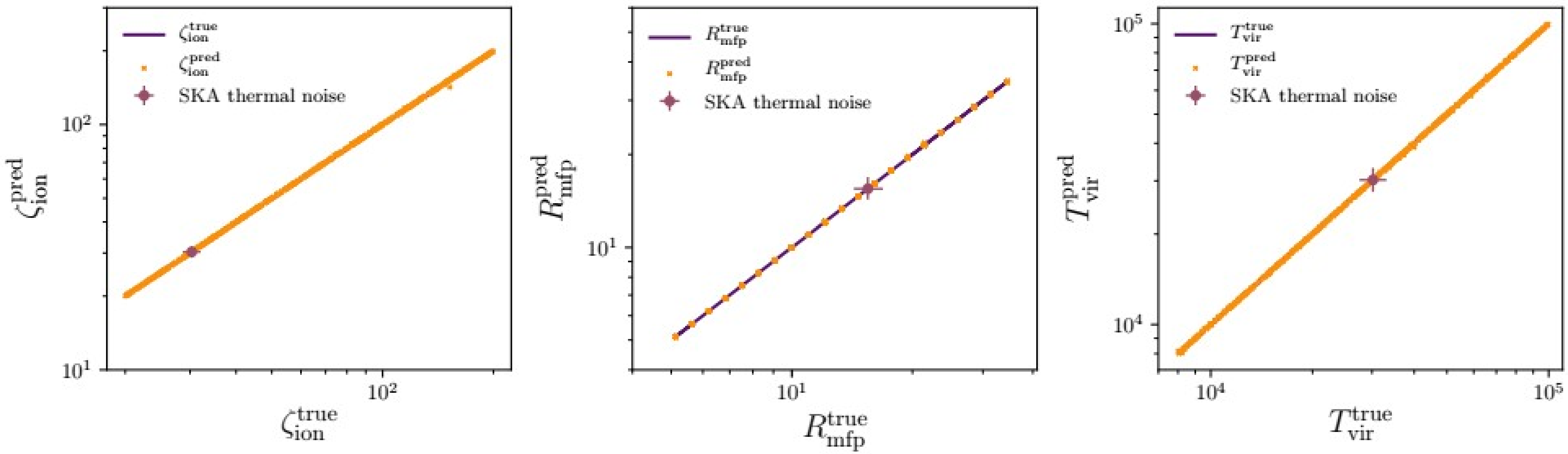
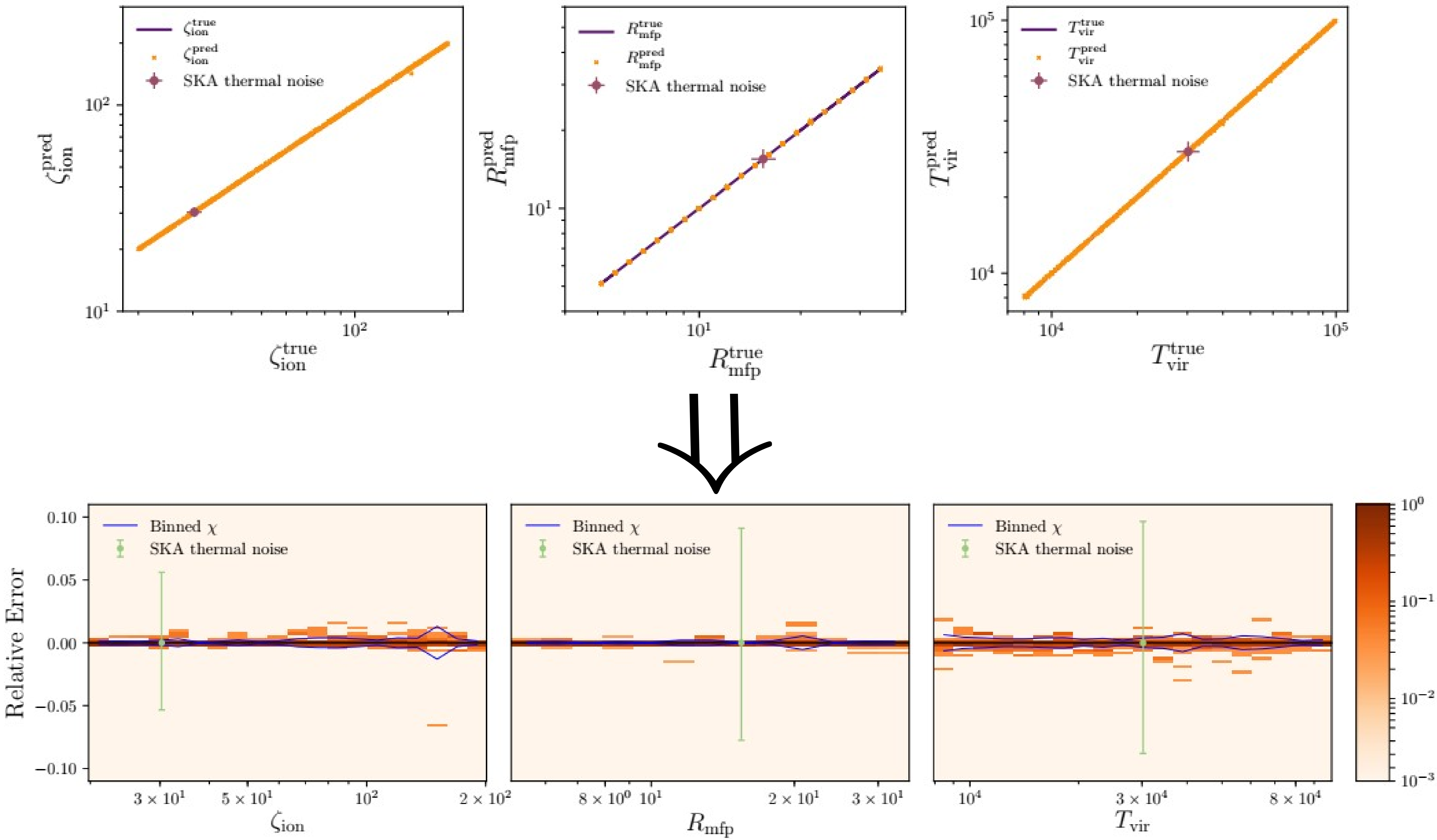
$\chi_{\mathrm{Kernel+Ridge\ Regression}}$	0.00676	0.01756	0.00091
$\chi_{\mathrm{Local\ Kernel+Ridge\ Regression}}$	0.00463	0.00722	0.00048

Local determination of the hyperparameters

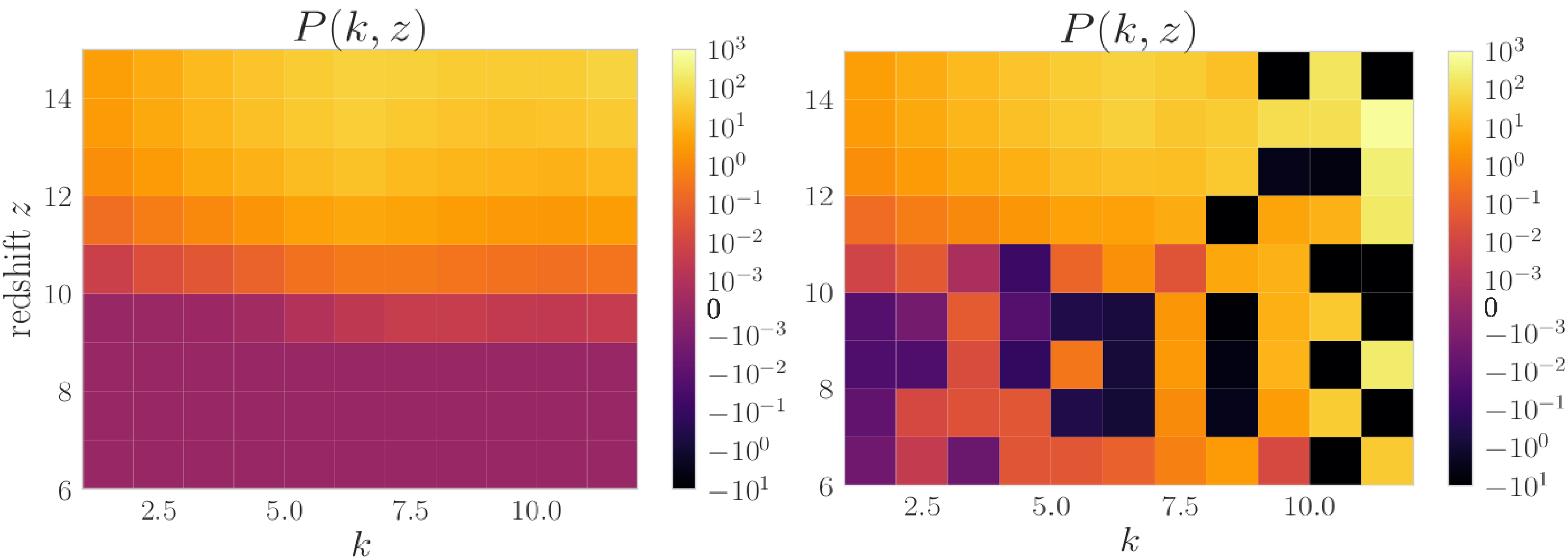


Local determination of the hyperparameters



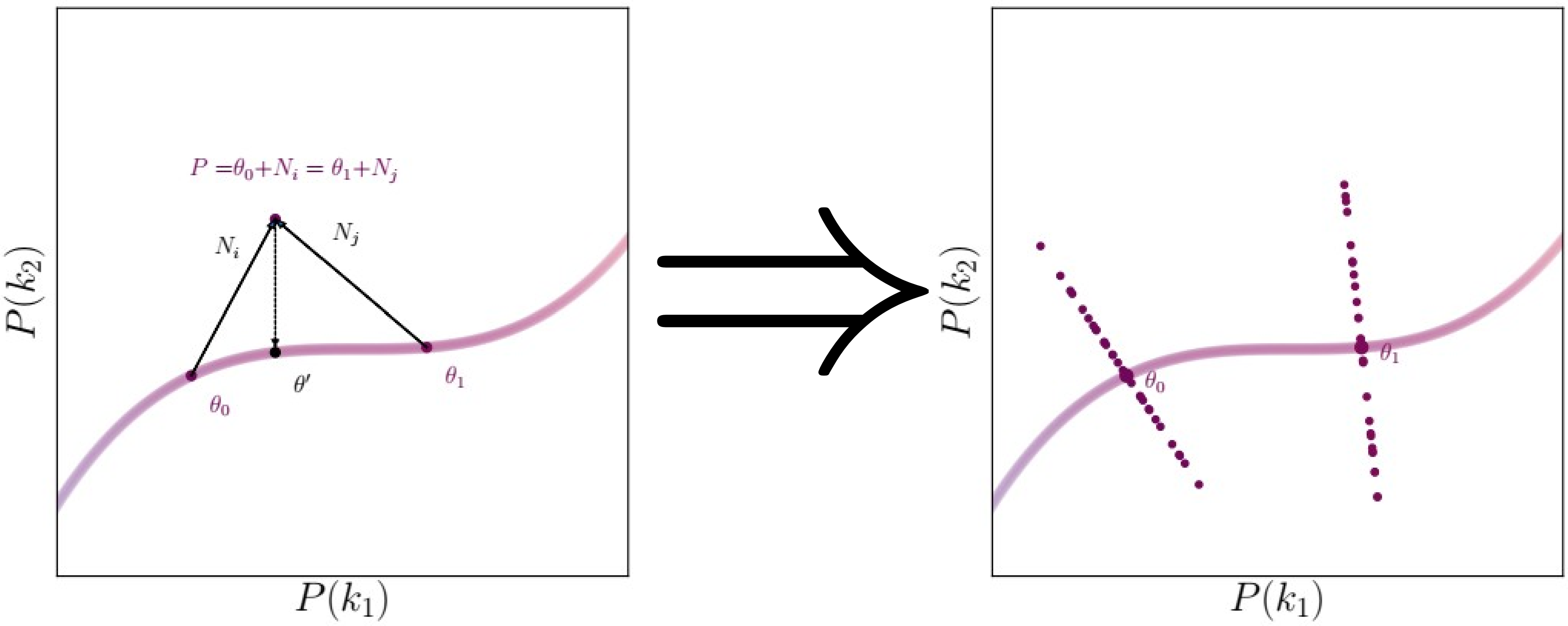
Reality : Noised signal

Effect of the noise on the input



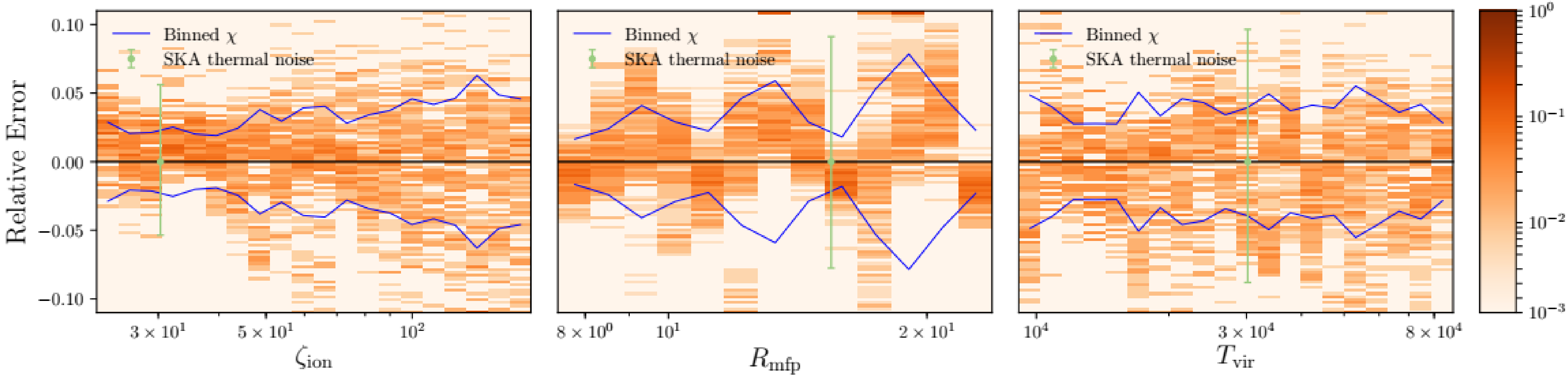
Data Preparation

- Use Signal to Noise Ratio
- Choice of correct answer (maximum likelihood)



Best Results to this day

With noise	$\chi_{\zeta_{\rm ion}}$	$\chi_{R_{\rm mfp}}$	$\chi_{log(T_{\rm vir})}$
Shimabukuro 2017	16.8×10^{-2}	17.2×10^{-2}	1.9×10^{-2}
Neural Network	3.70×10^{-2}	4.04×10^{-2}	0.41×10^{-2}



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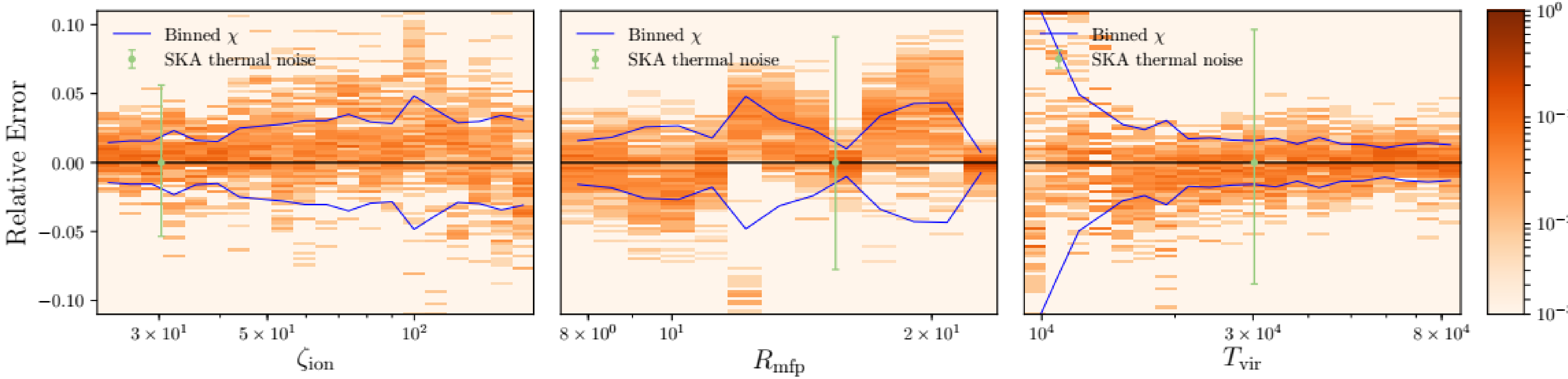
Global Ridge Kernel Regression	2.88×10^{-2}	2.84×10^{-2}	0.34×10^{-2}



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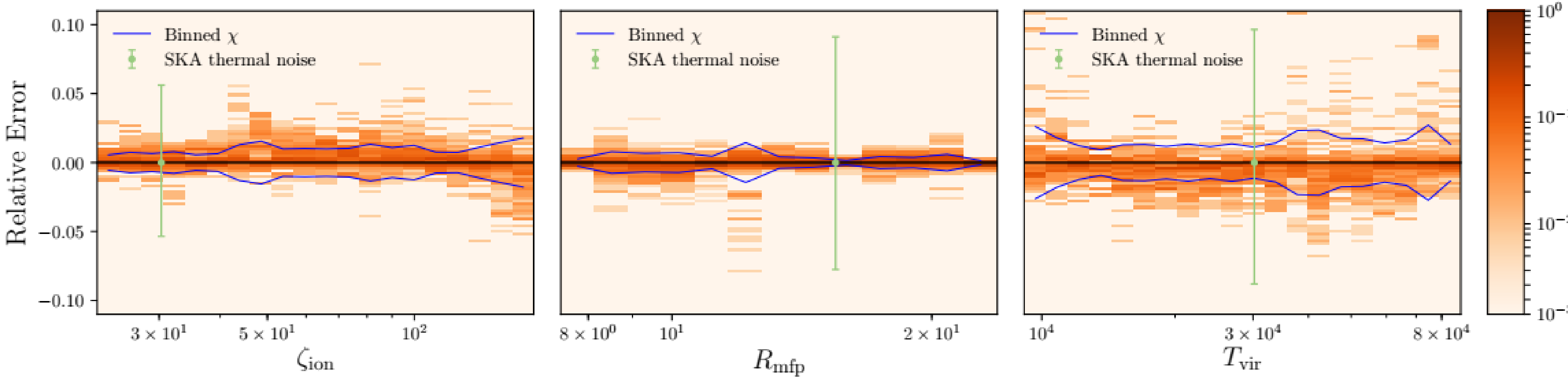
Neural Network	3.70×10^{-2}	4.04×10^{-2}	0.41×10^{-2}
Global Ridge Kernel Regression	2.88×10^{-2}	2.84×10^{-2}	0.34×10^{-2}
Local Ridge Kernel Regression	1.10×10^{-2}	0.60×10^{-2}	0.16×10^{-2}



Conclusion

– For Noise-free signals :

- Improvement of the quality of the predictions by a factor **50**
- Reconstruction error **<1 %** (SKA bayesian 1-sigma : **~5 %**)

– For Noised signals :

- Improvement of the quality of the predictions by a factor **10**
- Error on MaxLikelihood value **~1 %** (SKA bayesian 1-sigma : **~10 %**)

– What’s next :

- Apply to numerical simulation results
- Bayesian Neural Networks

Thank you for your attention